RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

FIRST YEAR

B.A./B.Sc. SECOND SEMESTER (January – June) 2015

Mid-Semester Examination, March 2015

Date : 21/03/2015

MATHEMATICS (General)

Time : 12 noon – 1 pm

Paper : II

Full Marks : 25

Group – A

Answer <u>any one</u> question :

1. Show that the condition that the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $\ell x+my=1$ is right angled is $(a+b)(a\ell^2+2h\ell m+bm^2)=0$. [5]

2. Find the transformation which transforms the equation $x^2+y^2-2x+14y+20=0$ into $x'^2+y'^2-30=0$. [5]

Group – B

3. **Answer <u>any one</u> :** [1×5]

   a) Determine the values of λ and μ for which the vectors $-3\vec{i}+4\vec{j}+\lambda\vec{k}$ and $\mu\vec{i}+8\vec{j}+6\vec{k}$ are collinear. [5]

   b) Show by vector method, that the perpendicular from the vertices of a triangle to the opposite sides are concurrent. [5]

4. **Answer <u>any one</u> :** [1×4]

   a) i) Define a monotone increasing sequence.

   ii) Give an example of a sequence which is monotone increasing but not strictly.

   iii) Give an example of a sequence which is convergent but neither monotone increasing nor monotone decreasing. [1+1+2]

   b) i) Define a bounded sequence.

   ii) Prove that every convergent sequence is bounded. [1+3]

5. **Answer <u>any one</u> :** [1×4]

   a) Find the values of a and b such that $\lim\limits_{x\to 0}\dfrac{x(1-a\cos x)+b\sin x}{x^3}=\dfrac{1}{3}$, assuming that L'Hospital's rule is applicable. [4]

   b) Show that the maximum value of $\left(\dfrac{1}{x}\right)^x$ is $e^{1/e}$. [4]

6. Answer <u>any one</u> : [1×3]

   a) Find $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\dfrac{n^2}{(n^2+r^2)^{3/2}}$. [3]

   b) If $I_n=\int_0^{\pi/2}\sin^n x\,dx\,(n\in\mathbb{Z}^+>1)$ prove that $I_n=\dfrac{n-1}{n}I_{n-2}$. Hence find the value of $\int_0^{\pi/2}\sin^5 x\,dx$. [2+1]

Group – C

7. **Answer <u>any one</u> :** [1×4]

a) i) Obtain the differential equation satisfied by the family of curves $x^2+y^2=1+(\lambda+y)^2$, where λ is a parameter. [2]

ii) Solve : $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\dfrac{x}{y}\right)dy=0$. [2]

b) i) Solve : $\log\left(\dfrac{dy}{dx}\right)=ax+by$. [2]

ii) Solve : $xdy-ydx=\sqrt{x^2+y^2}\,dx$. [2]

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